

MATH 502 HOMEWORK 1

Due Wednesday, January 31.

Problem 1. Let \mathcal{L} be a language with relation symbols \mathcal{R} , function symbols \mathcal{F} and constant symbols \mathcal{C} . Let $T_0 := C \cup \{v_1, v_2, \dots\}$ be the variables and constants of the language. Given T_k , let $T_{k+1} = \{f(t_1, \dots, t_{n_f}) \mid f \in \mathcal{F}, t_1, \dots, t_{n_f} \in T_k\}$. Set $T = \bigcup_k T_k$. Prove that T is the set of all terms.

Problem 2. Exercise 2.16.

Problem 3. Exercise 2.19.

Problem 4. Let M be an \mathcal{L} -structure and let $\sigma : V \rightarrow M$ be an assignment.

a) Suppose that $M \models_{\sigma} \exists v_1 \forall v_2 \phi$. Prove that $M \models_{\sigma} \forall v_2 \exists v_1 \phi$. Give an example showing that the converse is false.

b) Give an example of $\mathcal{L}, M, \sigma, \phi$ and ψ , such that $M \models_{\sigma} (\exists v_1 \phi) \wedge \psi$, but $M \not\models_{\sigma} \exists v_1 (\phi \wedge \psi)$.

Problem 5. Let $\mathcal{L} = \{E\}$, where E is a binary relation symbol. Define a theory T , such that $M \models T$ if and only if E^M is an equivalence relation and all equivalence classes are infinite.